

# System Identification in GSM/EDGE Receivers Using a Multi-Model Approach

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**Abstract**—Model order selection is an important element in system identification. It is well known that common model order selection methods such as Akaike's information criterion (AIC) and Bayesian's information criterion (BIC) neglect relevant information that is available in models of order different from the one chosen. In this paper the model order selection problem for receivers similar to those found in GSM and EDGE systems is reviewed briefly and is solved with a multi-model approach based on simultaneous consideration of several models. Two methods are evaluated; a multi-model noise suppression filter and multi-model soft combining. The algorithms are implemented and evaluated by means of simulations. The performance of each method is analyzed for GSM and EDGE receivers in a link level simulator. Simulation results show a significant improvement in performance at the cost of increased computational complexity for the multi-model approach.

**Index Terms**—System Identification, Multi-model, GSM, EDGE, Whitening, Equalizer

## I. INTRODUCTION

The benefits of using parametric models for system identification have been recognized for several decades in the area of artificial intelligence, pattern recognition, stochastic modeling, signal processing, and digital communications. The equalizers in modern digital communication receivers, such as GSM and EDGE, utilize discrete model structures, e.g., discrete time finite-impulse-response (FIR) filter and/or autoregressive (AR) processes. The parameters of these models, including the model order, are typically estimated adaptively to reflect the varying radio conditions [1]. Clearly, just like any other model parameter, the choice of model order will affect the system performance considerably and is therefore an important area of research. The more commonly used model order selection methods, including Akaike's information criterion (AIC) derived using Kullback-Leibler (KL) information and Bayesian's information criterion (BIC), are based on maximum-likelihood (ML) estimation principle (see e.g., [2]–[6]). BIC is preferred over AIC when  $N$  is small, where  $N$  is model order, but asymptotically they are equivalent, i.e., when  $N \rightarrow \infty$  or [6]. Unfortunately, AIC and BIC are prone to under-estimating and over-estimating the model order and are known to neglect relevant information that may be available in models of order different from the one chosen. To exploit this knowledge, a new composite method based on the simultaneous consideration of several models of different orders, the so-called multi-model approach has recently been introduced in [7]. In this work, we apply the

multi-model approach of [7] to equalizers similar to those found in GSM and EDGE receivers. We evaluate a two-step approach, where in the first step, we apply the multi-model approach to the noise model of [7] to estimate a hypothesized multi-model noise whitening filter. In the second step, we combine the soft-outputs, which are obtained by the soft-output Viterbi algorithm (SOVA) [9], from several parallel equalizers with different channel lengths. The idea of multi-model soft combining was first presented in [10] for multi-user detection problem with somewhat different approach and was later extended in [1] to get the multi-model soft outputs. The performance of the multi-model whitening filter and multi-model SOVA are compared against an adaptive length whitening filter and single model SOVA with channel length estimation, respectively. The evaluations are performed in a link-level simulator for different propagation models; static, rural, typical urban and hilly terrain at different terminal speeds of 1, 3, 50, 100 and 250 km/h. The simulator considers both the co-channel and adjacent channel interference. The simulation results for GSM/EDGE receivers with a single antenna branch and two antenna branches are presented. Results for two diversity combining schemes [11], Maximum Ratio Combining (MRC) and Interference Rejection Combining (IRC) are compared for the receivers having two antenna branches. The results show that the multi-model whitening filter provides low gain for the single branch receivers but there is significant gain in the two branch receivers. However, an equalizer with multi-model soft combining outperforms the ordinary adaptive length equalizer for both single and multiple branch receivers. This high gain is achieved at the cost of increased computational complexity.

## II. MULTI-MODEL SYSTEM IDENTIFICATION FOR GSM/EDGE RECEIVERS

### A. The Multi-model Approach

In most model selection processes, information criteria such as AIC and BIC, as mentioned above, are often used to select the model order for further processing. The simplified expression for the AIC rule is given as [6]

$$\min_{l \in [l_{\min}, l_{\max}]} [AIC = -2 \ln(\text{likelihood}) + 2l], \quad (1)$$

where  $l$  is the number of independently adjusted parameters, i.e., model order in our case. Model order using BIC can be obtained in the same way as AIC with simple approximation given in [6] as,

$$\min_{l \in [l_{\min}, l_{\max}]} [BIC = -2 \ln(\text{likelihood}) + (\ln N)l] \quad (2)$$

In general, it is hard to make a good estimate of the model order. When using single model selection we only choose one model order and neglect other possibilities. The multi-model approach [7] for model-order selection is designed to deal with this problem. In multi-model model-order selection the AIC or BIC is used to estimate the *a posteriori* probabilities associated to each model w.r.t. training data, and the output of several models can finally be combined using a weighting function. The multi model approach is investigated for BIC in [6], but it is also stated that these conclusion are applicable to AIC more or less directly. The AIC rule for multi-model selection can be obtained in the following way. Let  $y$  be the observed data vector. The *a posteriori* probability of data can be approximated as

$$p(y|M_l) \approx \alpha e^{-\frac{AIC_l}{2}}, \quad (3)$$

where  $\alpha$  is a constant, and  $M_l$  represents the hypothesis that a certain model with order  $l \in [0, L]$  has generated the data  $y$ . Assuming all the models are a priori equally probable, i.e.,  $P(M_l)$  is a constant, we can write the Bayesian rule as,

$$P(M_l|y) = \frac{p(y|M_l)P(M_l)}{p(y)}. \quad (4)$$

Using (3) in (4) and if  $\beta = \frac{\alpha P(M_l)}{p(y)}$  is constant. We get,

$$p(M_l|y) \approx \beta e^{-\frac{AIC_l}{2}}. \quad (5)$$

The constant in (5) can be obtained as,

$$\sum_{k=0}^L P(M_k|y) = \beta \sum_{k=0}^L e^{-\frac{AIC_k}{2}} = 1. \quad (6)$$

Using (6) in (5) gives the estimate of *a posteriori* probability of model  $M_l$  obtained by AIC as,

$$\hat{P}(M_l|y) = \begin{cases} \frac{e^{-\frac{AIC_l}{2}}}{\sum_{k=0}^L e^{-\frac{AIC_k}{2}}} & l = 0, 1, \dots, L \\ 0 & \text{Otherwise} \end{cases} \quad (7)$$

Hence, by means of *a posteriori* probability  $\hat{P}(M_l|y)$  the multi-model estimate of parameter vector  $\hat{\theta}_{mm}^L$  is given as (for details see [7]),

$$\hat{\theta}_{mm}^L = \hat{P}(M_0|y)\hat{\theta}^0 + \hat{P}(M_1|y)\hat{\theta}^1 + \dots + \hat{P}(M_L|y)\hat{\theta}^L. \quad (8)$$

### B. Multi-model Noise Suppression

Let  $Z_n$  be the  $n^{\text{th}}$  received data vector. After the synchronization and least square channel estimation [8] the system model of the received data vector can be interpreted as

$$Z'_n = \hat{H}S_n + W_n, \quad (9)$$

where  $W_n$  is a  $(d \times 1)$  residual vector representing noise and interference together, and  $(d \times d)$  matrix  $\hat{H}$  modeling channel transfer function is estimated by the channel estimator.

Typically one passes  $Z'_n$  through a whitening filter to make the interference temporally uncorrelated. The reference receiver of GSM/EDGE is shown in Fig.1. The system simulator models  $W_n$  as vector auto-regressive (VAR) process with model order  $l \in [0, L]$ . The difference equation of a VAR process of order  $L$  can be written as

$$W_n = -A_1 W_{n-1} - \dots - A_L W_{n-L} + \varepsilon_n, \quad (10)$$

where  $A_1, A_2, \dots, A_L$  are  $(d \times d)$  matrices of the VAR process and  $\varepsilon_n$  is an error vector with zero mean and covariance  $Q$ , i.e.,  $Q = E\{\varepsilon_n \varepsilon_n^H\}$ . The whitening filters in the reference receivers select the filter length  $l \in [0, L]$  adaptively using AIC and then estimate the filter coefficients using the Yule-Walker equation (see [12] and [13]). In this contribution, we replace the adaptive whitening filter with a multi-model whitening filter which estimates the filter coefficients for VAR models of every order, i.e.,  $l = 0, 1, \dots, L$  and weight them together according to their *a posteriori* probabilities. The block diagram of the modified whitening filter is shown in the Fig. 2. We compare the results of multi-model whitening filter against adaptive length selection based whitening filter in reference receivers. If the  $VAR_L$  model is driven by complex valued white Gaussian noise, then the estimated parameter vector  $\hat{\theta}^L$  will have  $L+1$  coefficient matrices, i.e.,  $\hat{\theta}^L = [\hat{A}_0 \hat{A}_1 \dots \hat{A}_L]$  each containing complex parameters. Considering that  $\hat{A}_0$  is identity matrix, the estimated parameter vector for VAR process of order using multi-model approach can be obtained using (8) as follows,

$$\hat{\theta}_{mm}^L = [\hat{A}_0 \ 0 \ \dots \ 0] \hat{P}(M_0|y) + [\hat{A}_0 \ \hat{A}_1 \ \dots \ 0] \hat{P}(M_1|y) + \dots + [\hat{A}_0 \ \hat{A}_1 \ \dots \ \hat{A}_L] \hat{P}(M_L|y). \quad (11)$$

The  $\hat{\theta}_{mm}^L$  always has  $L+1$  coefficient matrices where  $L$  is the highest permissible model order. The probability  $\hat{P}(M_l|y)$  in (8) is estimated by using the simplified  $AIC_l$  derived explicitly for the VAR process as

$$AIC_l = \log(|\hat{C}_l|) + d(\log \pi + 1) + \frac{2ld^2}{N}, \quad (12)$$

where  $\hat{C}_l$  is the covariance matrix. The multi-model coefficients  $\hat{\theta}_{mm}^L$  are then used as the coefficients of the multi-model whitening filter.

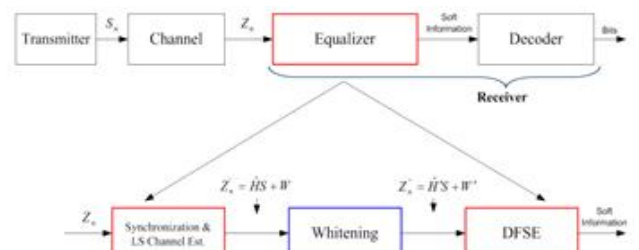


Fig.1. Block Diagram of Reference GSM/EDGE Receivers

### C. Multi-model Soft Combining

The Viterbi algorithm in our simulator keeps track of the most likely path sequences through the state trellis but it is modified to provide also the reliability information or soft value for each bit in the form of log-likelihood ratios (LLRs)

$$\gamma_k = \log \frac{P\{b_k = +1|Z_n\}}{P\{b_k = -1|Z_n\}} \quad (13)$$

whereas  $P\{b_k = -1|Z_n\}$  and  $P\{b_k = +1|Z_n\}$  are loglikelihood ratios (or *a posteriori* probabilities) with respect to received data  $Z_n$  that a certain information bit  $b_k$  is  $+1$  or  $-1$  is transmitted. The algorithm used to compute the softs is called Soft Output Viterbi Algorithm (SOVA) [9]. In the reference GSM and EDGE receivers the equalizer adapts to most likely channel length such that the mean squared error is minimized. It gives the soft decision  $\gamma_{k|l}$  for a certain information bit  $b_{k|l}$  conditioned on channel length  $l$  as (12), with the assumption that  $l \in [l_{min}, l_{max}]$  is true channel length estimate. If the estimated is not the true channel length then the soft decisions can be inappropriate and lead to high error probability. This problem can be solved by a multi-model methodology, by running several equalizers in parallel for all possible channel lengths and combining the output in an optimum way (for details see [1]). The block diagram for multi-model soft combining which we implement is shown in Fig. 3. We estimate the soft values for all possible channel lengths and combine them with the estimated model order *a posteriori* probabilities. The multi-model SOVA with model weighting can be written as,

$$\gamma_k = \log \frac{\sum_{l_{min}}^{l_{max}} \hat{P}(M_l|y) \frac{e^{\gamma_{k|l}}}{1 + e^{\gamma_{k|l}}}}{1 - \sum_{l_{min}}^{l_{max}} \hat{P}(M_l|y) \frac{e^{\gamma_{k|l}}}{1 + e^{\gamma_{k|l}}}} \quad (14)$$

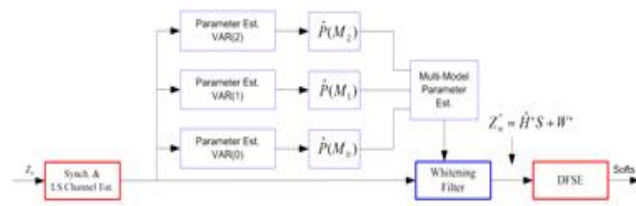


Fig.2 Block Diagram of Multi-Model Whitening Filter

The multi-model soft outputs calculated above are given to the decoder to obtain the data sequence. Since we are running more than one equalizer it is obvious that the complexity of the receivers will increase.

## III. TEST SCENARIOS AND SIMULATION PARAMETERS

The algorithms are evaluated for a number of test cases for the performance evaluation. These test cases include various types of propagation models, interference formats, modulation and coding schemes and receivers with single antenna or two antenna branches. The reference receivers are adaptive based on single model selection.

TABLE I. THE DATA RATE AND MODULATION TYPE FOR EACH OF MODULATION AND CODING SCHEMES MCS1-MCS9

Modulation & Coding Scheme (MCS)	Speed(Kbits/s/slot)	Modulation
MCS1	8.8	GMSK
MCS2	11.2	GMSK
MCS3	14.8	GMSK
MCS4	17.6	GMSK
MCS5	22.4	8PSK
MCS6	29.6	8PSK
MCS7	44.8	8PSK
MCS8	54.4	8PSK
MCS9	59.2	8PSK

### A. Modulation and Coding Schemes

The modulation and coding schemes tested are MCS1-MCS9 as listed in Table I and the results for MCS1, MCS3, MCS6, and MCS8 are presented in section IV.

### B. Propagation Models

The propagation models are defined in Table II according to [14]. Channel models including Hilly Terrain (HT), Rural Area (RA) and Typical Urban (TU) are analyzed. Terminals moving with low speed (3 km/h and 50 km/h) and high speed (100 km/h and 250 km/h) are tested with and without frequency hopping respectively.

### C. Receiver Types and Interference Formats

Receivers with one and two branches are tested for each of the above modulation and coding schemes. The Maximum Ratio Combining (MRC) and the Interference Rejection

TABLEII. PROPAGATION MODELS UNDER TEST

Propagation Model	Terminal Speed(Km/h)	Frequency Hopping
Static	1	No
Typical Urban (TU)	3	Yes
Typical Urban (TU)	50	Yes
Hilly Terrain (HT)	100	No
Rural Area (RA)	250	No

Combining (IRC) is taken into account for the receivers having two antenna branches. The multi-model algorithms evaluated both for the receivers without diversity and the receivers with diversity (MRC/IRC). All the above mentioned test cases are studied and the obtained results are summarized in Section IV. The results are achieved by simulating the different algorithms for 10,000 radio bursts. The simulations are run for both the interference limited and noise limited scenarios. In the interference limited scenarios Signal to Noise ratio (Eb/No) is set to 45 dB and the performance is evaluated for



TABLE III. MULTI-MODEL WHITENING FILTER WITH CO-CHANNEL AND CO-CHANNEL MULTIPLE INTERFERENCE

Receiver Type	Mode 1	Speed (Km/h)	Performance Gain for 10% BLER (dB)			
			MCS 1	MCS 3	MCS 6	MCS 8
No Diversity	TU	3	0.3	0.3	0.1	0
	TU	50	0.2	0.4	-0.1	0
	HT	100	0.2	0.3	0.3	0.5
	RA	250	0.5	0.4	0	0
2 Branches MRC	TU	3	5	3.4	4	3
	TU	50	4.5	3	4	2
	HT	100	1	1	0	0
	RA	250	4	2.5	1.5	0
Co-Channel Multiple Interference						
No Diversity	TU	3	0	0	0	0
		50	0	0	0	0
2 Branches MRC	TU	3	1.5	1	1	0.5
		50	1.5	1	1	0.5

different Carrier to Interference ratios (C/I). Several interference formats are investigated, e.g., co-channel interference, multiple co-channel interference, and the adjacent channel interference.

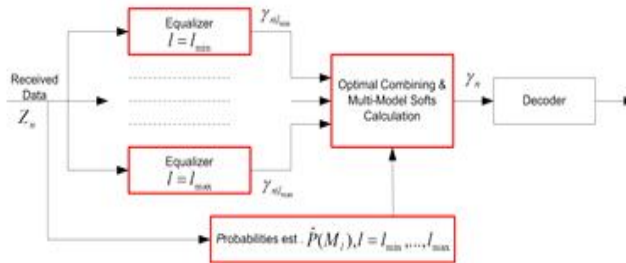


Fig.3. Block Diagram of Multi-Model Softs Combining

The noise limited scenarios (sensitivity tests) are tested for completeness where the C/I ratio is set to 1000 dB and the performance is evaluated for different Eb/No. For the sensitivity test it is assumed that there is no interference and the noise is AWGN.

#### IV. NUMERICAL EXAMPLES

Due to space limitation the simulation results are summarized only for 10% Block Error Rate (BLER) and are categorized according to different interference formats in the following sub sections.

##### A. Example 1: Multi-model Whitening Filter

First of all the model order selection is reviewed and the comparison is made between the classical AIC of (1) and the modified AIC of (11). From the results it is observed that the modified AIC is more appropriate choice for model order selection. Hence modified AIC is used to calculate the model weights. The adaptive filter's length varies from 1 to 3 depending upon the type of interference. Hence a multi-model

Whitening filter of length 3 is compared against the adaptive length whitening filter. The simulation results show that the multi-model noise suppression filter has better performance for both the single branch receiver and the two branches MRC based receiver (see Tables III and IV).

TABLE IV. MULTI-MODEL WHITENING FILTER WITH ADJACENT CHANNEL INTERFERENCE

Receiver Type	Model	Speed (Km/h)	Performance Gain for 10% BLER (dB)			
			MCS 1	MCS 3	MCS 6	MCS 8
No Diversity	TU	3	-0.6	-0.5	-1.1	-0.8
	TU	50	-1	-0.9	-1	-0.5
	HT	100	-0.7	-0.3	-0.8	0
	RA	250	-1	-0.7	-0.7	0
2 Branches MRC	TU	3	0.5	0.5	2.5	1
	TU	50	0.4	0.4	2.5	1
	HT	100	0	0.6	1.2	0.4
	RA	250	0	0	-1	-3

However, the multi-model noise suppression degrades the performance for the single branch receiver in the presence of adjacent channel interference; the reason for this degradation in performance could be due to under/over fitting the model order. The multi-model noise suppression filters show no gain in case of IRC because in IRC "the gain" has already been achieved with other algorithm, i.e., spatio-noise decorrelation.

##### B. Example 1: Multi-model Soft Combining

The multi-model soft outputs with model weighting are obtained by combining soft outputs from five parallel equalizers. As the adaptive equalizers under study select the channel length adaptively, from  $l_{min}$  taps up to a maximum of  $l_{max}$  taps, we modified these receivers such that they run equalizations for five channel lengths in parallel and combine the soft outputs obtained from each of these equalizers. These multi-model equalization based receivers are compared against the adaptive equalization based receivers. The detailed comparison of all these methods and relative gains for 10% BLER is summarized in Tables V and VI. The results show that the multi-model soft combining outperform the ordinary adaptive length equalization. A significant gain is obtained at the cost of increased computational complexity. The idea behind mixing multi-model soft values is to take all the possible channel lengths into account so that any information associated to certain channel length is not missed. To reduce the computational complexity, instead of considering all possible channel lengths one can think of combining soft values only from the most effective lengths, i.e., models having highest weights. Hence, for sake of comparison, from five parallel equalizers we select only two equalizers that carry highest weights and combine their softs with multi-model approach. When the results for five parallel equalizers are compared against two equalizers carrying highest *a posteriori* probabilities it is observed that difference in performance is

TABLE V. SOFT COMBINING WITH ADJACENT CHANNEL INTERFERENCE; FIVE PARALLEL EQUALIZERS

Receiver Type	Model	Speed (Km/h)	Performance Gain for 10% BLER (dB)			
			MCS 1	MCS 3	MCS 6	MCS 8
No Diversity	TU	3	3	2.8	1	1
	TU	50	2.9	3.1	1.2	1
	HT	100	4.2	4	2	0
	RA	250	3	3	1	0
2 Branches MRC	TU	3	5	5	2	1
	TU	50	5	5	2	1
	HT	100	3.5	3.5	2	0
	RA	250	5	4	2	1.5
2 Branches IRC	TU	3	2.5	2.5	2	1
	TU	50	2.5	2.5	2	1.5
	HT	100	4	4	3	3
	RA	250	0.8	0.9	1	0.5

relatively small. It is not possible to show all the comparison results due to lack of space however sample results are shown in Fig. 4. For complete results see [15].

TABLE VI. SOFT COMBINING WITH CO- CHANNEL INTERFERENCE; FIVE PARALLEL EQUALIZERS

Receiver Type	Mode 1	Speed (Km/h)	Performance Gain for 10% BLER (dB)			
			MCS 1	MCS 3	MCS 6	MCS 8
No Diversity	TU	3	0.5	0.3	0.8	0.5
	TU	50	0.4	0.4	0.7	0.5
	HT	100	0.9	1.1	0.5	0.2
	RA	250	0.2	0	0.5	0
2 Branches MRC	TU	3	0.4	0.6	1	0.3
	TU	50	0.2	0.5	0.5	0.3
	HT	100	0.6	1	0.8	-1
	RA	250	0.5	0.5	0.5	0.5
2 Branches IRC	TU	3	3	3	2	1
	TU	50	3	3	2	1
	HT	100	1	1	1	0
	RA	250	0	0	0	1
Co-Channel Multiple Interference						
No Diversity	TU	3	0.7	1	0.8	0
		50	0.5	0.6	0.8	0
2 Branches MRC	TU	3	0.7	0.8	0.8	0.5
		50	0.7	1	0.8	0.5
2 Branches IRC	TU	3	1	2	1	0.5
		50	1	2	1	0.5

## V. SUMMARY AND CONCLUSIONS

The paper focuses on multi-model system identification in GSM/EDGE receivers. This approach is adapted to reduce the overall identification error. For this purpose the related information from each model is used in proportion to their weights or *a posteriori* probabilities. Akaike's information criterion (AIC) is used to obtain the weight of each model. The multi model identification based equalizers are implemented for receivers with one antenna and two antenna branches (MRC/IRC) respectively. It is observed that the performance is gained at the cost of increased complexity.

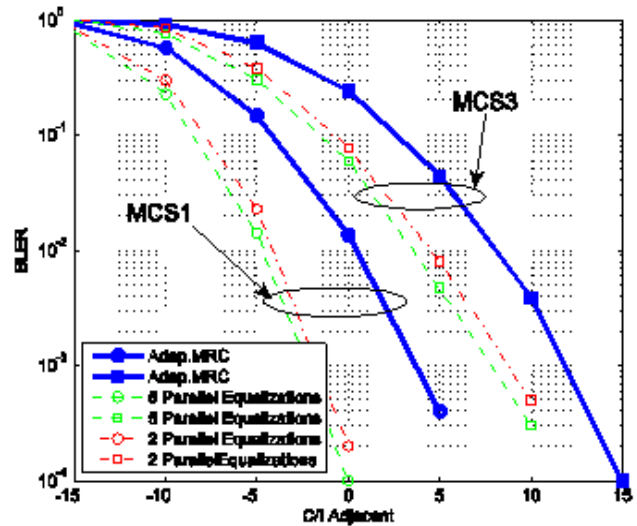


Fig.4. BLER comparison plot for different soft combining methods; receiver is MRC based with GMSK modulation tested for rural scenario with terminal speed of 250 Km/h

The center of attention in this paper was the performance evaluation against single model identification based reference receivers. The performance is evaluated by means of simulations under different modulation and coding schemes (MCSs), numerous interference formats and propagation models. The simulation results show that the multi-model noise suppression filters have better performance for single branch receiver and the two branches MRC based receiver. Whereas the IRC based receivers and the multi-model noise suppression filters have almost the same performance gain. An adaptive equalizer is suboptimal if the estimated channel length is incorrect. On the other hand a multi-model equalizer takes care of all permissible channel lengths according to their weights thus chances of losing relative information by adaptively selecting an inappropriate channel length can be reduced. From the simulation it is observed that the multi-model soft combining outperformed the adaptive equalizer. An effective gain has been achieved for all the propagation models and interference formats with an increase in computational complexity. Finally, it is suggested that the computational complexity can be reduced if we combine the softs only from the most appropriate channels instead of all permissible channels. Therefore soft outputs from two parallel equalizers are combined. Simulation results show that the computational complexity has been decreased with almost the same performance as with the five parallel equalizers.

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